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DYNAMIC STABILITY OF A CONIC SHELL SUPPORTED ALONG ONE EDGE AND LOADED WITH AXIALLY SYMMETRICAL PRESSURE

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This paper contains a study of parametric oscillations of a circular conical shell freely supported along one edge. The derived theoretical correlations are limited by the results of the experiments performed.

Let us consider a thin-walled, circular, conical shell freely supported along one edge and loaded with external pressure p depending upon time t. We relate the median surface of the shell to coordinates s and Θ as is shown in Figure 1.

To investigate the parametric oscillations of the shell we resort to the equations of Lagrange:

$$\frac{d}{dt}\left(\frac{K}{\delta a_i}\right) - \frac{K}{\delta a_i} + \frac{(U + \ )}{\delta a_i} = 0 \qquad (1)$$

wherein K is the kinetic energy of motion of the system; U internal potential energy of the system; Π the potential of external forces; a_i' derivative of the general coordinate a_i with respect to time.

Let us undertake the determination of K, U, and Π appearing in the Lagrange equations.

For a thin-walled conical shell the internal potential energy U is expressed by the formular [1]:

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$$U = \frac{Eh^3}{24(1-\mu^2)}\iint\left[\varkappa_1^2 + \varkappa_2^2 + 2\mu\varkappa_1\varkappa_2 + 2(1-\mu)\tau^2\right] r\,ds\,d\theta +$$
$$+ \frac{Eh}{2(1-\mu^2)}\iint\left[\varepsilon_1^2 + \varepsilon_2^2 + 2\mu\varepsilon_1\varepsilon_2 + \frac{1-\mu}{2}\omega^2\right] r\,ds\,d\theta \qquad (1)$$

wherein

$$\varkappa_1 = \frac{\partial^2 w}{\partial s^2}, \qquad \varkappa_2 = \frac{1}{r^2}\left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta}\cos\alpha\right) + \frac{\partial w}{r\,\partial s}\sin\alpha$$
$$\tau = \frac{1}{r}\left(\frac{\partial^2 w}{\partial\theta\,\partial s} - \frac{\partial w}{r\,\partial\theta}\sin\alpha\right) + \frac{\cos\alpha}{r}\left(\frac{\partial v}{\partial s} - \frac{v}{r}\sin\alpha\right)$$
$$\varepsilon_1 = \frac{\partial u}{\partial s} + \frac{1}{2}\left(\frac{\partial v}{\partial s}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2$$
$$\varepsilon_2 = \frac{\partial v}{r\,\partial\theta} - \frac{w}{r}\cos\alpha + \frac{u}{r}\sin\alpha + \frac{1}{2}\left(\frac{\partial w}{r\,\partial\theta} + \frac{v}{r}\cos\alpha\right)^2 + \frac{1}{2}\left(\frac{\partial u}{r\,\partial\theta} - \frac{v}{r}\sin\alpha\right)^2$$
$$\omega = \frac{\partial v}{\partial s} - \frac{v}{s} + \frac{\partial u}{r\,\partial\theta}$$

Herein u, v, w are the displacements of the point of median surface of the shell, respectively, in the direction of generatrix, the peripheral direction and the direction of the normal.

Kinetic energy of the shell is expressed by the following evident correlation

$$K = \frac{1}{2}\iint m_0\left[\left(\frac{du}{dt}\right)^2 + \left(\frac{dv}{dt}\right)^2 + \left(\frac{dw}{dt}\right)^2\right] r\,d\theta\,ds$$

The potential of external forces we define as

$$\Pi = -\int p\,dV$$

wherein dv is the element of change in volume circumscribed by the median surface of the shell.

For a conical shell [2]:

$$dV = \frac{1}{2}\left[2w + w(\varepsilon_1 + \varepsilon_2) - \frac{v^2}{r}\cos\alpha - \frac{\partial w}{\partial s}u - \frac{\partial w}{r\,\partial\theta}v\right] r\,d\theta\,ds$$

Let us now consider the functions of displacements u, v, and w by means of which we have expressed all the quantities appearing in the equation of Lagrange.

A conic shell freely supported along one edge can undergo diformation without elongation and shift of median surface. Let us assume that such a deformation is sustained by the shell during parametric oscillations. In such a case the functions of displacements can be determined from the system of equations

$$
\begin{gathered}
\varepsilon_1 = \frac{\partial u}{\partial s} + \frac{1}{2}\left(\frac{\partial v}{\partial s}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2 = 0 \\
\varepsilon_2 = \frac{\partial v}{r\,\partial\theta} - \frac{w}{r}\cos\alpha + \frac{u}{r}\sin\alpha + \frac{1}{2}\left(\frac{\partial w}{r\,\partial\theta} + \frac{v}{r}\cos\alpha\right)^2 + \frac{1}{2}\left(\frac{\partial u}{r\,\partial\theta} + \frac{v}{r}\sin\alpha\right)^2 = 0 \\
\omega = \frac{\partial v}{\partial s} - \frac{v}{s} + \frac{\partial u}{r\,\partial\theta} = 0
\end{gathered}
\tag{6}
$$

On solving this system approximately with an accuracy to a^2 and taking into account that $w = 0$ with $s = s_1$, we get [2]:

$$
u = u_0 + u_1 + u_2, \quad v = v_1, \quad w_0 + w_1 + w_2. \tag{7}
$$

wherein

$$
\begin{aligned}
u_1 &= a s_1 \frac{\cos\alpha \sin\alpha}{n^2 - \sin^2\alpha} \cos n\theta \\
v_1 &= \frac{a}{n}\left[(s - s_1)\cos\alpha - s_1 \frac{\cos\alpha \sin\alpha}{n^2 - \sin^2\alpha}\right] \sin n\theta \\
w_1 &= a(s - s_1)\cos n\theta \,\frac{1}{1} \\
u_2 &= -\frac{a^2}{2}(s - s_1)\left(\frac{\cos^2\alpha}{n^2}\sin^2 n\theta + \cos^2 n\theta\right) \\
w_2 &= u_2 \operatorname{tg}\alpha + \frac{a^2}{2}\,\frac{(n^2 - \cos^2\alpha)^2}{n^2\cos\alpha \cdot \sin\alpha}\,\frac{(s - s_1)^2}{s}\sin^2 n\theta
\end{aligned}
\tag{8}
$$

herein a is an arbitrary free parameter.

The appearance of the deformed shell is shown in Figure 2.

Displacements u_0 and w_0, corresponding to the symmetrical form of deformation, we will not calculate, since with the selected functions of diplacements they have no effect on the changes of potential and kinetic energy of the shell during parametric oscillations.

Let us calculate K, U, and Π. Since it is sufficient to [illegible] the potential [illegible] with an [illegible] it follows that u_2 and w_2 will appear only in the expression of the potential of external forces.

On substituting the function of displacements (8) in (3) and integrating over the entire surface, we get

$$K = \frac{1}{8}(\dot{a})^2 m_0 w_1^2 \frac{(k-1)^3}{\sin^3\alpha}\left[k^2 - 0.25(k+1)^2\right]\left(1 + \frac{\cos^2\alpha}{n^2}\right) \qquad \left(k = \frac{r_2}{r_1}\right) \tag{9}$$

After integration of expression (2) we get

wherein $$U = U_0 + \frac{a^2}{2}\pi\frac{Eh^3}{12(1-\mu^2)}X_1 \tag{10}$$

$$X_1 = \frac{\lambda^2}{(k-1)^3}\left[(n^2-1)^2 \ln k - 2(n^2-1)(n^2-\cos^2\alpha)\left(1-\frac{1}{k}\right) + \right.$$
$$\left. + \frac{1}{2}(n^2-\cos^2\alpha)^2\left(1-\frac{1}{k^2}\right)\right] + \frac{\lambda}{k^2}(k+1)\frac{(1-\mu)(n^2-\cos^2\alpha)^2}{n^2} \qquad \left(\lambda = \frac{l}{r_1}\right)$$

With small angles α it is more convenient to calculate U by expanding X_1 to a series by power of $(K-1)$.

$$X_1 = \lambda(n^2-1)^2\left[\frac{1}{3k^3} + \frac{k-1}{4k^4} + \frac{(k-1)^2}{5k^5} + \dots\right] \lambda(n^2-1)\frac{k-1}{k^3} +$$
$$+ \frac{1}{2}\frac{(k-1)^2(k+1)}{\lambda k^4} + \frac{\lambda}{k^2}(k+1)\frac{(1-\mu)(n^2-\cos^2\alpha)}{n^2} \tag{11}$$

On integrating the expression (4) with respect to θ within the limits from 0 to 2π and discarding the small terms, we get

$$\Pi = -\frac{a^2}{2}\pi\frac{(n^2-\cos^2\alpha)}{\cos\alpha}\int_{s_1}^{s_2} p(s-s_1)^2\,ds \tag{12}$$

We will assume that the pulsation component of pressure varies along the generatrix in proportion to the variation of its static component, i.e., we assume the following law of variations of pressure loading the shell:

$$p = p_0 f(s)\left[1 + \frac{\Delta p}{p_0}\psi(t)\right] \tag{13}$$

wherein Δp is a quantity proportional to the amplitude of pressure pulsation.

Let us expand the function $f(s)$ to a Taylor series by $(s - s_1)$

$$f(s) = \sum_{i=0}^{\infty} c_i \left(\frac{s - s_1}{l}\right)^i$$

In such a case on taking the integral, in the expression (12) from s_1 to s_2; we get

$$\Pi = -\frac{a^2}{2}\pi p_0 \frac{r_1^3}{\cos\alpha} X_2 \left[1 + \frac{\Delta p}{p_0}\psi(t)\right] \quad (14)$$

wherein

$$X_2 = (n^2 - \cos^2\alpha)\lambda^3 \sum_{i=0}^{\infty} \frac{c_i}{i+3}$$

We revert now to the Lagrange equation. In our case the potential and kinetic energy of the shell depend only upon a single free parameter a, which is the one we will take as the general coordinate. Then on substituting the calculated values of K, U, and Π in the equation of Lagrange, we get, after simple transpositions, the final expression of the equation of parametric oscillations of the conic shell

$$\ddot{a} + \Omega^2[1 - \varepsilon\psi(t)]a = 0 \quad (15)$$

wherein

$$\varepsilon = \frac{\Delta p}{p^* - p_0}$$

$$\Omega^2 = \frac{3\sin^2\alpha \operatorname{tg}\alpha\, a^2 X_2}{m_0 r_1 (k-1)^3 [k^2 - 0.25(k+1)^2](n^2 + \cos^2\alpha)}(p^* - p_0) \quad (16)$$

$$p^* = \frac{Eh^3\cos\alpha}{12 r_1^3 (1 - \mu^2)} \frac{X_1}{X_2}$$

In particular, for a cylindrical shell

$$\Omega^2 = \frac{n^2(n^2 - 1)}{r m_0 (n^2 + 1)}(p^* - p_0)$$

$$p^* = \frac{Eh^3(n^2 - 1)}{12 r^3 (1 - \mu^2)}\left[1 + \frac{6(1 - \mu) r^2}{n^2 l^2}\right] \quad (17)$$

The quantity ε is called coefficient of pulsation. The coefficient Ω, appearing in equation (15), is the frequency of natural oscillations of the shell, while n^* coincides in form with the expression of the critical pressure [2]; only n corresponding to the minimum value of Ω, generally speaking, is not equal to the number of waves n corresponding to the minimum critical pressure.

The derived equation of parametric oscillations of a conic shell is a differential equation having a periodic coefficient of the Mathieu type, which has been thoroughly investigated with $\psi(t) = \sin \theta t$. In this instance, in addition to the solution $a = 0$ which corresponds to an absence of transversal oscillations, it has also a solution of the form

$$a = e^{\lambda_1 t} f(t)$$

With definite correlations between the coefficients of the equation, the exponent λ_1 can have a positive real portion and in such a case a is found to be increasing without bounds with time. It is this instance which corresponds to a dynamic instability of the shell.

Investigation of the Mathieu equation shows that there exist a number of regions of dynamic instability. With small values of the parameter ε the boundaries of the first critical region are defined by the correlation [3]

$$2\Omega\sqrt{1 - \frac{1}{2}\varepsilon - \frac{7}{32}\varepsilon^2 - \frac{55}{512}\varepsilon^3} < \theta < 2\Omega\sqrt{1 + \frac{1}{2}\varepsilon + \frac{7}{32}\varepsilon^2 + \frac{55}{512}\varepsilon^3} \qquad (18)$$

and the boundaries of the second by the correlation

$$\Omega\sqrt{1 - \frac{1}{12}\varepsilon^2} < \theta < \Omega\sqrt{1 + \frac{5}{12}\varepsilon^2}, \qquad (19)$$

and so forth.

But as concerns shells, of substantial importance is only the first region of instability, since the resonance corresponding to the second and the subsequent regions is very difficult to induce even under laboratory conditions due to the damping processes.

From formula (16) it is apparent that the shell has an infinite number of natural frequency values corresponding to the different n. Therefore within each critical region there exist moreover an infinite number of dynamic instability zones.

From the practical standpoint it may be of importance to know the boundaries of the dynamic instability zone determined by the lowest frequency of natural oscillations. In the instance of fastening of the conic shell considered herein, the lowest frequency corresponds to n = 2. Figure 3 shows the graph of variation of minimum frequencies of natural oscillations of the shell with p = 0, wherein on the axis of ordinates is plotted the non-dimensional parameter

$$\phi = \Omega \sqrt{r_1^4 \; m_0 / Eh^3}$$

All the formulas derived in the present paper hold true also in the case of shells subjected to the action of internal pressure; only when they are applied must the minus sign which precedes the value of the static component of pressure be replaced with a plus sign.

It is not difficult to see that the formulas for Ω and p^* of a ring [1, 4] follow from the correlations derived in the present paper:

$$\Omega^2 = \frac{n^2(n^2 - 1)}{rm_0(n^2 + 1)} (p^* - p), \quad p^* = \frac{(n^2 - 1) EI}{r^3} \qquad (20)$$

For the determination of the frequencies of natural oscillations

[illegible]

[illegible]

1. [illegible] cm; [illegible]

2. α = ? r_1 = 2.9 cm; h = 0.015 cm

The length of the shells was varied within the limits of 27 to 15 cm; p was made equal to zero.

Figure 4 shows on oscillogram of the recorded parametric oscillations of one of these shells ($\alpha = 6^\circ$; $r_1 = 4.175$ cm; $h = 0.015$ cm; $\ell = 27$ cm).

The top curve is a sinusoid of alternating current having a frequency of 50 cycles; the middle curve -- that of the variation of pressure pulsation frequency -- and the bottom curve is the recording of the shell oscillations.

The oscillogram shows clearly that the shell passes from induced oscillations, of a frequency equal to that of pressure pulsation, to parametric oscillations of a frequency equal to about one half the frequency of pulsation.

In the graph (Figure 5) the results of experiments are compared with the theoretical correlations calculated by means of the formulas derived in the present paper; the frequency of natural oscillations of the shell is taken as equal to the mean frequency of the area of parametric resonance.

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FIGURES

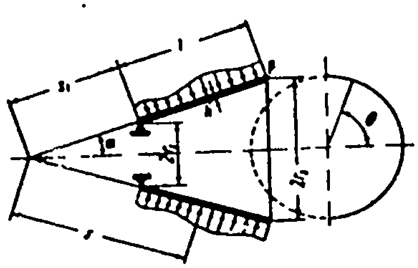

Figure 1

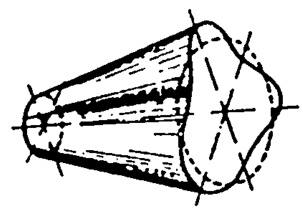

Figure 2

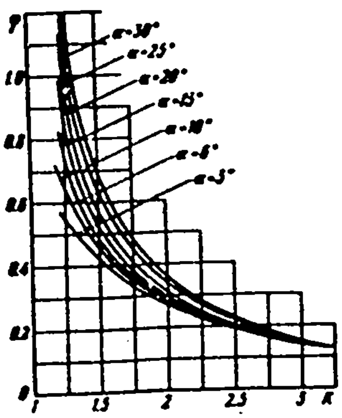

Figure 3

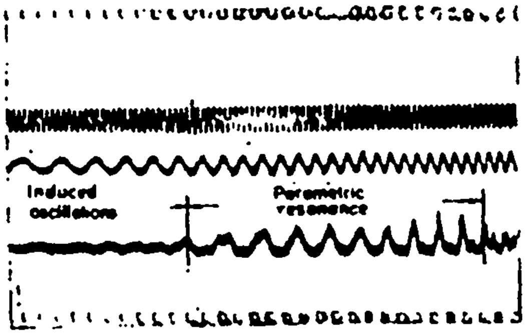

Figure 4

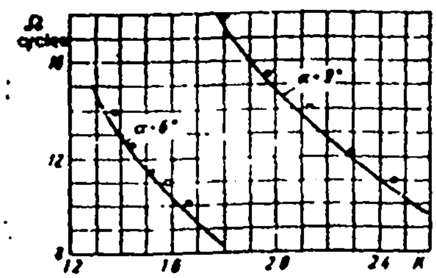

Figure 5